Test 2 Review Answers

(1) $f'(x)=\frac{|x|}{x}$ $\quad f''(x)=\frac{x\frac{d}{dx}|x|-|x|\frac{d}{dx}x}{x^2}=\frac{x\cdot\frac{|x|}{x}-|x|}{x^2}=0$ (2) 3 (3) Vertical tangent, cusp/corner, discontinuity

(4) x-intercept (5) True (6) $-\csc^2 x$ (7) $\frac{-10}{t^6}$ (8) 0 (9) False, only for diff'able fns.

(10) $f'(x)=\lim_{z\to x}\frac{\frac{1}{3-z}-\frac{1}{3-x}}{z-x}=\lim_{z\to x}\frac{3-x-(3-z)}{(z-x)(3-z)(3-x)}=\lim_{z\to x}\frac{z-x}{(z-x)(3-z)(3-x)}=\lim_{z\to x}\frac{1}{(3-z)(3-x)}=\frac{1}{(3-x)^2}$

(11) See book (12) $\frac{1-5x^3}{2\sqrt{x}(1+x^3)^2}$ (13) Note $x\sqrt[3]{x}=x^{4/3}$ so $\frac{d}{dx}(x\sqrt[3]{x})=\frac{4}{3}x^{1/3}$, $\quad y'=x^3\sec^2 x+3x^2\tan x+\frac{4}{3}\sqrt[3]{x}$

(14) Chain twice $y'=6\sin 3x\cos 3x$ (15) $y'=\frac{2\cos 4x}{\sqrt{1+\sin 4x}}$ (16) $y'=\frac{x^2(3-2x^2)}{(1-x^2)^{3/2}}$

(17) Simplify first $y=\frac{x}{x^3}-\frac{4x^5}{x^3}=x^{-2}-4x^2$ So $y'=-2x^{-3}-8x=\frac{-2}{x^3}-8x$

(18) Find where $f'(x)=0$ $\quad y'=2\sec x\tan x-\sec^2 x\overset{\text{set}}{=}0$ $\quad \sec x(2\tan x-\sec x)=0\Rightarrow$

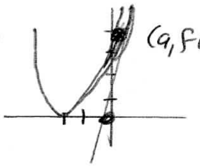
$\sec x=0$ (never) or $2\tan x-\sec x=0\Rightarrow\frac{2\sin x}{\cos x}-\frac{1}{\cos x}=0\Rightarrow 2\sin x-1=0$ $\quad \sin x=\frac{1}{2}\Rightarrow x=\frac{\pi}{6},\frac{5\pi}{6}$

So points: $\left(\frac{\pi}{6},f\left(\frac{\pi}{6}\right)\right)\Rightarrow\left(\frac{\pi}{6},\sqrt{3}\right)$ $\quad \left(\frac{5\pi}{6},f\left(\frac{5\pi}{6}\right)\right)\Rightarrow\left(\frac{5\pi}{6},-\sqrt{3}\right)$

(19) need $m_{tan}=\left.\frac{dy}{dx}\right|_{(4,3)}$ $\quad$ Implicit diff $\Rightarrow 2x+2yy'=0\Rightarrow y'=\frac{-x}{y}$ $\quad \left.\frac{dy}{dx}\right|_{(4,3)}=\frac{-4}{3}$

eqn. $y-3=\frac{-4}{3}(x-4)$

(20)



Let $(a,f(a))=(a,(a+2)^2)$ be the point of tangency. Need to find a.

From calculus, we know $m_{tan}=f'(a)=2(a+2)$

From algebra, slope through $(0,0)$ and $(a,(a+2)^2)$ is $\frac{(a+2)^2}{a}$ $\Big\}$ set equal and solve for a

$\Rightarrow a=-2,2$ So points are $(-2,0)$ $(2,16)$

(21) Acceleration is $s''(t)=12t^2-24t$ So Accel $=0$ when $12t^2-24t=0$

$12t(t-2)=0$ $\quad t=0,2$ sec

(22) Since $(1,1)$ satisfies both equations (check) then $(1,1)$ is an intersection pt.

For $y^2=x^3$, by implicit diff we can show $\left.\frac{dy}{dx}\right|_{(1,1)}=\frac{3}{2}$

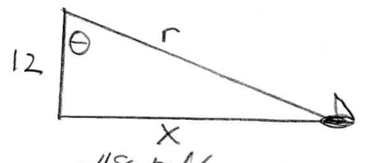
and for $2x^2+3y^2=5$ $\quad \left.\frac{dy}{dx}\right|_{(1,1)}=-\frac{2}{3}$ $\quad\Big\}$ Since slopes are negative reciprocals, tangent lines are perpendicular.

(23) $-2^{13}\sin 2x$

(24) Let $f(x)=\sqrt{x}$ $\quad x=25$ $\quad f(x+\Delta x)=f(x)+\Delta y\approx f(x)+dy$

$dy=\frac{1}{2\sqrt{x}}dx$ $\quad \Delta x=-.2$ $\quad \sqrt{24.8}\approx\sqrt{25}+\frac{1}{2\sqrt{25}}(-.2)=5-.02=4.98$

(25)



want speed of boat $=\left.\frac{dx}{dt}\right|_{r=13}$ and $\left.\frac{d\theta}{dt}\right|_{r=13}$ $\quad$ Know $\frac{dr}{dt}=-4$ $\quad x^2+144=r^2$

$2x\frac{dx}{dt}=2r\frac{dr}{dt}\Rightarrow\frac{dx}{dt}=\frac{r}{x}\frac{dr}{dt}$

$\frac{r}{12}=\sec\theta$

$\left.\frac{d\theta}{dt}\right|_{r=13}=\frac{-48}{65}$ rad/sec $\longleftarrow$ $\frac{1}{12}\frac{dr}{dt}=\sec\theta\tan\theta\frac{d\theta}{dt}$ $\quad \left.\frac{dx}{dt}\right|_{r=13}=\frac{13}{5}(-4)=\frac{-52}{5}$ ft/sec