Note: There were no 7.5 problems or graphs here but there might be on yours so study it.

MATH 5B – SAMPLE TEST 1
(Chapter 7)

100 POINTS

Solutions

NAME: ____________________________

No scratch paper. Show all work clearly and completely with proper presentation on this exam. No credit will be given for solutions if work is not shown (except on the first ten problems where it is not necessary to show work). No graphing calculator allowed.

** INVERSE HYPERBOLICS FORMULA PAGE ALLOWED - NO GRAPHING CALCULATORS**

Fill in the blanks. (2 points each)

(1) $\frac{d}{dx}\underbrace{\ln 5}_{\text{constant}} =$ __0__

(2) $\int e^{4x}dx =$ __$\frac{1}{4}e^{4x}+C$__

(3) $\cos^{-1}(-1/2) =$ __$\frac{2\pi}{3}$__ (exact)

(4) $\lim_{x\to\infty} \tan^{-1}x =$ __$\frac{\pi}{2}$__

(5) $\int \tan x\, dx =$ __$\ln|\sec x|+C$__

CIRCLE T FOR TRUE, F FOR FALSE.(2 pts each)

T F (6) $\ln(x+y) = \ln x + \ln y$. — F

T F (7) If $\ln x = 5$, then $x = e^5$. — T

T F (8) $\frac{d}{dx}\cosh x = -\sinh x$ — F

T F (9) $\lim_{x\to\infty} e^{-x} = 0$ — T

T F (10) $\frac{d}{dx}3^x = 3^x \ln 3$ — T

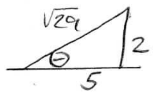
(11) Simplify each of the following (exactly...i.e. no calculator) : (3 pts each)

(a) cosh(ln 3)

$$\frac{e^{\ln 3}+e^{-\ln 3}}{2} = \frac{3+\frac{1}{3}}{2} = \frac{5}{3}$$

(b) $\sin(\tan^{-1}(2/5)) = \sin\theta = \frac{2}{\sqrt{29}}$

$\theta=\tan^{-1}\frac{2}{5}$ means $\tan\theta=\frac{2}{5}$ and $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$



(12) Find the inverse function: $f(x) = \frac{e^x}{e^x+1}$ (5 points)

$$y = \frac{e^x}{e^x+1}$$

switch x & y

$$x = \frac{e^y}{e^y+1}$$

This is now the inverse, solve for y

$$x(e^y+1) = e^y$$
$$xe^y + x = e^y$$
$$xe^y - e^y = -x$$
$$e^y(x-1) = -x$$
$$e^y = \frac{-x}{x-1} = \frac{x}{1-x}$$
$$y = \ln\left(\frac{x}{1-x}\right)$$
$$f^{-1}(x) = \ln\left(\frac{x}{1-x}\right)$$

(13) Given $f(x) = \ln x + \tan^{-1}(x)$

(a) Find $\frac{d}{dx}\left[f^{-1}(x)\right]_{x=\pi/4}$ (5 points)

$$\frac{d}{dx}\left[f^{-1}(x)\right]\Big|_{x=\pi/4} = \frac{1}{f'(f^{-1}(\frac{\pi}{4}))}$$

$$= \frac{1}{3/2}$$

$$= 2/3$$

; where $f'(x) = \frac{1}{x} + \frac{1}{1+x^2}$

$f^{-1}(\frac{\pi}{4}) = 1$ (see below)

So $f'(f^{-1}(\pi/4)) = f'(1) = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}$

$f^{-1}(\frac{\pi}{4})$: $\frac{\pi}{4} = \ln x + \tan^{-1}(x)$

$x = 1$ (by inspection)

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(14) Find the derivative of each of the following functions and simplify: (4 pts each)

(a) $y = \ln\left(\frac{1}{x}\right) + \frac{1}{\ln x}$

Can write y as

$y = \ln 1 - \ln x + (\ln x)^{-1}$

$y = -\ln x + (\ln x)^{-1}$

So $y' = -\frac{1}{x} - (\ln x)^{-2}\frac{1}{x}$

$y' = -\frac{1}{x} - \frac{1}{x(\ln x)^2}$

(b) $y = \tanh^{-1}(\sin x)$

$\frac{d}{dx}\tanh^{-1}x = \frac{1}{1-x^2}$

so $\frac{d}{dx}\tanh^{-1}(\sin x) = \frac{1}{1-(\sin x)^2}\frac{d}{dx}\sin x$

$y' = \frac{1}{1-\sin^2 x}\cos x$

$y' = \frac{1}{\cos^2 x}\cos x = \frac{1}{\cos x} = \sec x$

(c) $y = \sec^{-1}(\sqrt{x})$

$\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$

So

$y' = \frac{1}{\sqrt{x}\sqrt{x-1}}\frac{d}{dx}(\sqrt{x})$

$y' = \frac{1}{\sqrt{x}\sqrt{x-1}}\frac{1}{2\sqrt{x}}$

$y' = \frac{1}{2x\sqrt{x-1}}$

(d) $y = (\sin x)^x$

$|y| = |(\sin x)^x|$

$\ln|y| = \ln|(\sin x)^x| = \ln|\sin x|^x = x\ln|\sin x|$

& differentiate $\frac{d}{dx}$

$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{\sin x}\cos x + \ln|\sin x|$

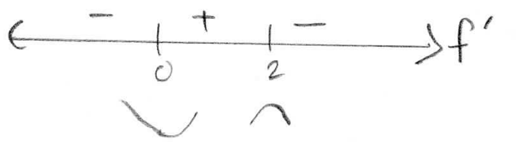
$\frac{dy}{dx} = y(x\cot x + \ln|\sin x|)$

$\frac{dy}{dx} = (\sin x)^x(x\cot x + \ln|\sin x|)$

(15) Find the local extrema for $f(x) = x^2 e^{-x}$. (6 pts)

$$f'(x) = -x^2 e^{-x} + 2x e^{-x}$$
$$= -x e^{-x}(x-2)$$

Crit #s when $x = 0, 2$



local max at $(2, 4e^{-2})$
local min at $(0, 0)$

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(16) Find the area of the region under the curve $y = \frac{9}{9+x^2}$; $-3 \le x \le 3$

(6 pts)

$$A = \int_{-3}^{3} \frac{9}{9+x^2}\,dx$$

$$A = 9\int_{-3}^{3} \frac{1}{9+x^2}\,dx$$

Using $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

$$= 9 \quad \frac{1}{3}\tan^{-1}\frac{x}{3}\Big]_{-3}^{3}$$

$$= 3\left(\tan^{-1}1 - \tan^{-1}(-1)\right)$$

$$= 3\left(\frac{\pi}{4} - -\frac{\pi}{4}\right)$$

$$= \frac{3\pi}{2}$$

(17) Find the value of each of the following integrals (simplify answers): (5 pts each)

(a) $\displaystyle\int_1^2 \frac{5}{2-3x}dx$

$u = 2-3x$

$du = -3dx$

$$= \int_{-1}^{-4} \frac{5}{u}\frac{dx}{-3}$$

$$= -\frac{5}{3}\int_{-1}^{-4}\frac{1}{u}du$$

$$= -\frac{5}{3}\ln|u|\Big]_{-1}^{-4} = -\frac{5}{3}(\ln 4)$$

(b) $\displaystyle\int \frac{3t}{e^{t^2}}dt$

$u = t^2$

$du = 2t\,dt$

$$= \frac{3}{2}\int\frac{1}{e^u}du$$

$$= \frac{3}{2}\int e^{-u}du$$

$$= -\frac{3}{2}e^{-u}+C$$

$$= -\frac{3}{2}e^{-t^2}+C$$

(c) $\displaystyle\int_0^{1/\sqrt{6}} \frac{x}{\sqrt{1-9x^4}}dx$

$u = 3x^2$

$du = 6x\,dx$

$$= \frac{1}{6}\int_0^{1/2}\frac{1}{\sqrt{1-u^2}}du$$

$$= \frac{1}{6}\sin^{-1}u\Big]_0^{1/2}$$

$$= \frac{1}{6}\sin^{-1}\frac{1}{2} = \frac{1}{6}\frac{\pi}{6} = \frac{\pi}{36}$$

(d) $\displaystyle\int \frac{dx}{x\sqrt{(\ln x)^2-1}}$

$u = \ln x$

$du = \frac{1}{x}dx$

$$\int\frac{1}{\sqrt{u^2-1}}du = \cosh^{-1}u + C$$

$$= \cosh^{-1}(\ln x)+C$$

(18) What are the coordinates of the point on the curve $y = e^{2x}$ at which the tangent is parallel to the line $y = 4x+1$? (4 points)

$m=4$

need $y'=4$

$2e^{2x}=4$

$e^{2x}=2$

$2x=\ln 2$

$x=\frac{1}{2}\ln 2$

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(19) Find each of the following limits: (4 pts each)

(a) $\lim_{x\to\infty}\left(x^3e^{-x^2}\right) = \lim_{x\to\infty}\frac{x^3}{e^{x^2}} \overset{L}{=} \lim_{x\to\infty}\frac{3x^2}{2xe^{x^2}} = \lim_{x\to\infty}\frac{3x}{2e^{x^2}} = \lim_{x\to\infty}\frac{3}{4xe^{x^2}} = 0$

$\infty\cdot 0$ $\quad \frac{\infty}{\infty}$ $\quad \frac{\infty}{\infty}$

(b) $\lim_{x\to 0}\left(\frac{\cos x}{x^2}\right) = \infty$

$\frac{1}{0}$ That is not indeterminate, do not apply L'Hospitals

(c) $\lim_{x\to 0^+}\left(\frac{1}{\ln(x+1)}-\frac{1}{x}\right) = \lim_{x\to 0^+}\left(\frac{x-\ln(x+1)}{x\ln(x+1)}\right)$

$\infty-\infty$ $\quad \frac{0}{0}$

$= \lim_{x\to 0^+}\frac{\left(1-\frac{1}{x+1}\right)}{x\frac{1}{x+1}+\ln(x+1)}$

$= \lim_{x\to 0^+}\frac{x+1-1}{x+(x+1)\ln(x+1)}$

$= \lim_{x\to 0^+}\frac{x}{x+(x+1)\ln(x+1)}$

$\frac{0}{0}$

$= \lim_{x\to 0^+}\frac{1}{1+(x+1)\frac{1}{x+1}+\ln(x+1)} = \lim_{x\to 0^+}\frac{1}{2+\ln(x+1)} = \frac{1}{2}$