Section Example: Compute $\underbrace{T, N, B, K}_{\text{at } t=1}$. Find equation of normal plane when $t=1$

$\vec{r} = \langle 6t, 3\sqrt{2}t^2, 2t^3 \rangle$

* $\vec{r}\,' = \langle 6, 6\sqrt{2}t, 6t^2 \rangle = 6\langle 1, \sqrt{2}t, t^2 \rangle$

$\|\vec{r}\,'(t)\| = 6\sqrt{1+2t^2+t^4} = 6\sqrt{(1+t^2)^2} = 6(1+t^2)$

$\vec{T} = \frac{1}{\|\vec{r}\,'\|}\vec{r}\,' = \frac{1}{6(1+t^2)} 6\langle 1, \sqrt{2}t, t^2 \rangle = \boxed{(1+t^2)^{-1}\langle 1, \sqrt{2}t, t^2 \rangle = \vec{T}(t)}$

* $\vec{T}\,'(t) = -2t(1+t^2)^{-2}\langle 1, \sqrt{2}t, t^2 \rangle + (1+t^2)^{-1}\langle 0, \sqrt{2}, 2t \rangle$

$= (1+t^2)^{-2}\left[-2t\langle 1, \sqrt{2}t, t^2 \rangle + (1+t^2)\langle 0, \sqrt{2}, 2t\right]$

$= (1+t^2)^{-2}\left[\langle -2t, -2\sqrt{2}t^2 + \sqrt{2}(1+t^2), -2t^3+2t+2t^3 \rangle\right]$

$= (1+t^2)^{-2}\langle -2t, \sqrt{2}(1-t^2), 2t \rangle$

$\|\vec{T}\,'(t)\| = (1+t^2)^{-2}\sqrt{4t^2+2(1-t^2)^2+4t^2}$

$= (1+t^2)^{-2}\sqrt{4t^2+2-4t^2+2t^4+4t^2}$

$= (1+t^2)^{-2}\sqrt{2(1+2t^2+t^4}$

$= (1+t^2)^{-2}\sqrt{2(1+t^2)^2} = (1+t^2)^{-2}\sqrt{2}(1+t^2)$

$= \sqrt{2}(1+t^2)^{-1}$

$\vec{N}(t) = \frac{\vec{T}\,'}{\|\vec{T}\,'\|} = \frac{1+t^2}{\sqrt{2}}(1+t^2)^{-2}\langle -2t, \sqrt{2}(1-t^2), 2t \rangle$

$\boxed{\vec{N}(t) = \frac{1}{\sqrt{2}(1+t^2)}\langle -2t, \sqrt{2}(1-t^2), 2t \rangle}$

* $\vec{B} = \vec{T} \times \vec{N} = (1+t^2)^{-1}\left(\frac{1}{\sqrt{2}(1+t^2)}\right)\left[\langle 1, \sqrt{2}t, t^2 \rangle \times \langle -2t, \sqrt{2}(1-t^2), 2t \rangle\right.$

$= \frac{1}{\sqrt{2}(1+t^2)^2}\langle \sqrt{2}t^2(1+t^2), -2t(1+t^2), \sqrt{2}(1+t^2) \rangle$

$\boxed{\vec{B} = \frac{1}{\sqrt{2}(1+t^2)}\langle \sqrt{2}t^2, -2t, \sqrt{2} \rangle}$

$K = \frac{\|\vec{T}\,'\|}{\|\vec{r}\,'\|} = \frac{\sqrt{2}(1+t^2)^{-1}}{6(1+t^2)} = \boxed{\frac{\sqrt{2}}{6(1+t^2)^2}}$

At $t=1$ at point $\vec{r}(1) = \langle 6, 3\sqrt{2}, 2\rangle$

$$\vec{T}(1) = \frac{1}{2}\langle 1, \sqrt{2}, 1\rangle = \left\langle \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2}\right\rangle$$

$$\vec{N}(1) = \frac{1}{2\sqrt{2}}\langle -2, 0, 2\rangle = \left\langle -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right\rangle$$

$$\vec{B}(1) = \frac{1}{2\sqrt{2}}\langle \sqrt{2}, -2, \sqrt{2}\rangle = \left\langle \frac{1}{2}, \frac{-1}{\sqrt{2}}, \frac{1}{2}\right\rangle$$

Notice - all unit vecters and all mutually orthogonal

$$K(1) = \frac{\sqrt{2}}{6(2)^2} = \frac{\sqrt{2}}{24}$$

For normal plane

point $(6, 3\sqrt{2}, 2)$

$\vec{n}$ in direction of $\vec{T}$. can use $\vec{r}'(t) = 6\langle 1, \sqrt{2}t, t^2\rangle$

or even better $\langle 1, \sqrt{2}t, t^2\rangle$ at $t=1$

$\vec{n} = \langle 1, \sqrt{2}, 1\rangle$

plane $(x-6) + \sqrt{2}(y - 3\sqrt{2}) + (z-2) = 0$