**7B Test 2 Sample**
**Unit 2**

Fill in the blanks.

(1) True or False: $\cos^2\theta = 1 - \sin^2\theta$ ___true___ .

(2) cos (x - y) = ___cosx cosy + sinx siny___

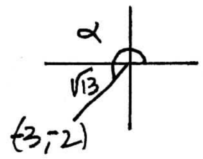
(3) $\sin(\theta/2) =$ ___$\pm\sqrt{\frac{1-\cos\theta}{2}}$___

(4) Write as a product: cos(6x)-cos(2x) ___$-2\sin 2x \sin 4x$___ . (Product ⟷ sum identities will be given)

(5) $\sin 2\theta =$ ___$2\sin\theta\cos\theta$___

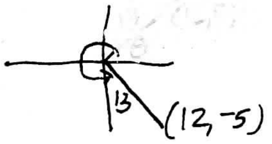
(6) Given $\tan\alpha = 2/3$, $\alpha$ in the third quadrant, and $\cos\theta = 12/13$, $\frac{3\pi}{2} < \theta < 2\pi$

Find:



a) $\sin(\alpha - \theta) = \sin\alpha\cos\theta - \cos\alpha\sin\theta$

$= \frac{-2}{\sqrt{13}}\frac{12}{13} - \frac{-3}{\sqrt{13}}\frac{5}{13} = \frac{-39}{13\sqrt{13}} = \frac{-3}{\sqrt{13}}$



b) $\cos(\theta/2) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$= -\sqrt{\frac{1+12/13}{2}} = \frac{-5}{\sqrt{26}}$

since $\frac{3\pi}{2} < \theta < 2\pi$

$\frac{3\pi}{4} < \frac{\theta}{2} < \pi$

$\frac{\theta}{2} \in$ Ⓘ$\mathrm{II}$ so $\cos\frac{\theta}{2}$ is negative

c) $\tan(2\alpha) = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\cdot 2/3}{1-\frac{4}{9}} = \frac{\frac{4}{3}}{1-\frac{4}{9}} = \frac{12}{5}$

(7) Using identities, find the exact value of:

(a) sin 108° cos 63° - cos 108° sin 63° = ___$\frac{\sqrt{2}}{2}$___

$= \sin(108° - 63°)$

$= \sin(45°)$

$= \frac{\sqrt{2}}{2}$

(b) $\cos(-\pi/12)$ ___$\frac{\sqrt{2}+\sqrt{6}}{4}$___

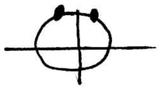
$\cos\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \frac{\sqrt{2}}{2}\frac{1}{2} + \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}$

(8) Solve for θ exactly (in radians, $0 \le \theta < 2\pi$).

(a) sin θ = 0.7



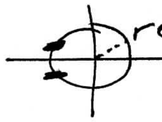
$\theta = \sin^{-1}0.7,\ \pi - \sin^{-1}(0.7)$

(b) tan θ = -4

ref $= \tan^{-1}4$

$\pi - \tan^{-1}4,\ 2\pi - \tan^{-1}4$

(c) cos θ = -5/6



ref $\cos^{-1}\frac{5}{6}$

$\pi - \cos^{-1}\frac{5}{6},\ \pi + \cos^{-1}\frac{5}{6}$

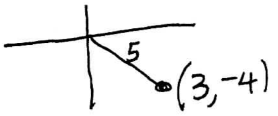
other answers possible, but they should be equivalent to ~~these~~ these. Check with calculator.

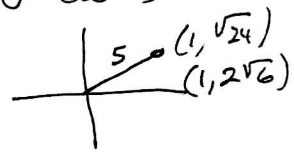
(9) Simplify $\sin\left(\tan^{-1}\left(\frac{-4}{3}\right)-\cos^{-1}\left(\frac{1}{5}\right)\right) = \sin(\alpha-\theta) = \sin\alpha\cos\theta - \cos\alpha\sin\theta$

$\alpha$ $\theta$

$$= \frac{-4}{5}\,\frac{1}{5} - \frac{3}{5}\,\frac{2\sqrt{6}}{5}$$

$\alpha = \tan^{-1}\left(-\frac{4}{3}\right)$ $\theta = \cos^{-1} 1/5$




$$= \frac{-4-6\sqrt{6}}{25}$$

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(10) Verify the following identity. Presentation counts.

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

RHS: $\frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\tan\theta}{\sec^2\theta} = \frac{2\,\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}} = 2\sin\theta\cos\theta = \sin 2\theta = LHS$

Therefore $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

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(11) Solve for $0 \le x < 2\pi$: $\sqrt{3}\tan(2x) + 1 = 0$

$\tan 2x = -\frac{1}{\sqrt{3}}$

$2x = \frac{5\pi}{6} + \pi k$

$x = \frac{5\pi}{12} + \frac{\pi}{2}k$

$x = \frac{5\pi}{12} + \frac{6\pi}{12}k$

In $[0, 2\pi)$

$x = \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}$

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(12) Find all solutions: $4\cos\left(\frac{x}{3}\right) = -4$

$\cos\left(\frac{x}{3}\right) = -1$

$\frac{x}{3} = \pi + 2\pi k$

$x = 3\pi + 6\pi k$

(14) $2\sin x - \sqrt{3} = 0$

$\sin x = \frac{\sqrt{3}}{2}$

$x = \begin{cases} \frac{\pi}{3} + 2\pi k \\ \frac{2\pi}{3} + 2\pi k \end{cases}$

(15) $4\cos^2 x - 2 = 0$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4} + \frac{\pi}{2}k$

| SOLVE the following equations: $0 \le x < 2\pi$ (8 points each) |
|---|

(16) $8 - 6\sin^2 x = 7\cos x$

$8 - 6(1 - \cos^2 x) = 7\cos x$

$8 - 6 + 6\cos^2 x = 7\cos x$

$6\cos^2 x - 7\cos x + 2 = 0$

$(3\cos x - 2)(2\cos x - 1) = 0$

$\cos x = \frac{2}{3}$ $\quad$ $\cos x = \frac{1}{2}$

$x = \cos^{-1}\frac{2}{3},\ 2\pi - \cos^{-1}\frac{2}{3},\ \frac{\pi}{3},\ \frac{5\pi}{3}$

(17) $\frac{1}{2}\cos t \sin(2t) = \sin t$

$\frac{1}{2}\cos t\ 2\sin t\cos t = \sin t$

$\sin t\cos^2 t - \sin t = 0$

$\sin t(\cos^2 t - 1) = 0$

$\sin t = 0$ $\quad$ $\cos^2 t = 1$

$\cos t = \pm 1$

$t = 0, \pi$