

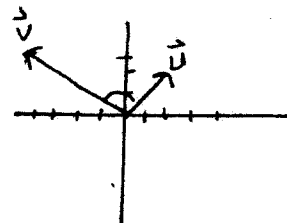
100 points

Show all work neatly. EXACT answers unless specified.

(1) Given the vectors $u = 2i + 2j$ and $v = -4i + 3j$, find the following:

a) $\|u\| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\frac{2\sqrt{2}}{\underline{\hspace{2cm}}}$



b) $u + v$

$\underline{\langle -2, 5 \rangle \text{ or } -2i + 5j}$

c) $u \cdot v$

$\underline{-2}$

d) The angle between u and v

$\cos \theta = \frac{u \cdot v}{\|u\| \|v\|} = \frac{-2}{10\sqrt{2}} = \frac{-1}{5\sqrt{2}}$

$\theta = \cos^{-1}\left(\frac{-1}{5\sqrt{2}}\right) \approx 98^\circ$ reasonable \uparrow

e) The direction angle of v (exact)

$\tan \theta = \frac{-3}{4}$ θ in Quad II

$\tan^{-1}\left(-\frac{3}{4}\right) \approx -36.9^\circ$

$\theta = 180^\circ + \tan^{-1}\left(-\frac{3}{4}\right) \approx 143.1^\circ$

f) Find a value for b such that $\langle b, 2 \rangle$ is orthogonal to v

$\underline{b = 3/2}$

$\langle b, 2 \rangle \cdot \langle -4, 3 \rangle = 0 \quad -4b + 6 = 0$

g) Find a value for c such that $\langle 8, c \rangle$ is parallel to v

$\underline{c = -6}$

parallel if constant multiple

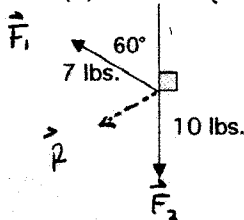
h) If \vec{PQ} is a representative of v where $P = (3, -1)$, find the coordinates of point Q .

$Q = (q_1, q_2) \quad \vec{PQ} = \langle q_1 - 3, q_2 + 1 \rangle = \langle -4, 3 \rangle \quad Q = (-1, 2)$

$q_1 - 3 = -4 \quad q_2 + 1 = 3$

$-2\vec{v} = -2\langle -4, 3 \rangle = \langle 8, -6 \rangle$
 \uparrow
 c

(2) Two forces act on an object as shown. Find the magnitude and the direction of the resultant.



(exact and approx.)

(10 pts)

$\vec{F}_1 = \langle 7 \cos 150^\circ, 7 \sin 150^\circ \rangle = \langle -\frac{7\sqrt{3}}{2}, \frac{7}{2} \rangle$

$\vec{F}_2 = \langle 0, -10 \rangle$

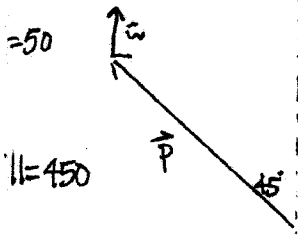
$\vec{R} = \vec{F}_1 + \vec{F}_2 = \langle -\frac{7\sqrt{3}}{2}, \frac{7}{2} - 10 \rangle = \langle -\frac{7\sqrt{3}}{2}, -\frac{13}{2} \rangle$ (reasonable)

$\|\vec{R}\| = \sqrt{\left(-\frac{7\sqrt{3}}{2}\right)^2 + \left(-\frac{13}{2}\right)^2} = \sqrt{\frac{147 + 169}{4}} = \sqrt{\frac{316}{4}} = \sqrt{79} \approx 8.9 \text{ lbs}$

$\tan \theta = \frac{-13/2}{-7\sqrt{3}/2} = \frac{13}{7\sqrt{3}}$, θ in Quad III

$\theta = \tan^{-1}\left(\frac{13}{7\sqrt{3}}\right) + 180^\circ \approx 227^\circ$

- (3) An airplane is traveling at a constant airspeed of 450 mph in the direction $N45^\circ W$. If wind is blowing directly northward at a rate of 50 mph, what is the actual speed and direction of the airplane?



$$\vec{R} = \vec{p} + \vec{w}$$

$$= \langle -225\sqrt{2}, 225\sqrt{2} + 50 \rangle$$

$$\vec{p} = \langle 450 \cos(135^\circ), 450 \sin(135^\circ) \rangle$$

$$= \langle -225\sqrt{2}, 225\sqrt{2} \rangle$$

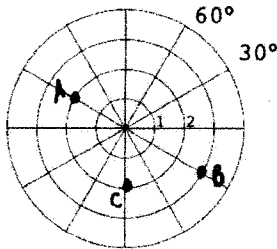
$$\vec{w} = \langle 0, 50 \rangle$$

$$\|\vec{R}\| = \sqrt{(-225\sqrt{2})^2 + (225\sqrt{2} + 50)^2}$$

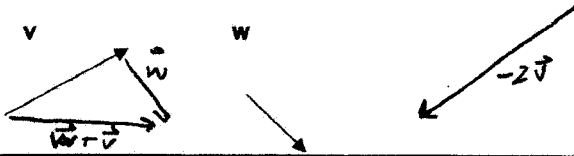
$$\approx 486 \text{ mph}$$

$$\tan \theta = \frac{225\sqrt{2} + 50}{-225\sqrt{2}} \quad \theta = \tan^{-1}\left(\frac{225\sqrt{2} + 50}{-225\sqrt{2}}\right) + 180^\circ \approx 130.8^\circ$$

- (4) On the axes below, plot (and label) the polar points A(2, 150°), B(3, $-\pi/6$), C(-2, $\pi/2$) (3pts)



- (5) Given the vectors \mathbf{w} and \mathbf{v} below, find $\mathbf{w} + \mathbf{v}$ and $-2\mathbf{v}$.



- (6) Given the point $(5, 7\pi/4)$ in polar coordinates, find the rectangular representation.

$$x = r \cos \theta = 5 \cos \frac{7\pi}{4} = 5 \frac{\sqrt{2}}{2}$$

$$y = r \sin \theta = 5 \sin \frac{7\pi}{4} = 5 \left(-\frac{\sqrt{2}}{2}\right)$$

$$\left(\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)$$

- (7) Given the point $(-1, \sqrt{3})$ in rectangular coordinates, find two different polar representations;

one with $r > 0$, the other with $r < 0$.

$$r^2 = x^2 + y^2 = 1 + 3 = 4 \quad r = \pm 2$$

$$\tan \theta = \frac{\sqrt{3}}{-1}, \text{ Quad 2}$$

$$\left(2, \frac{2\pi}{3}\right)$$

$$\left(-2, -\frac{\pi}{3}\right)$$

other answers possible.

- (8) Convert to rectangular coordinates: $r \sec \theta = 4$

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$$r \cdot \frac{1}{\cos \theta} = 4$$

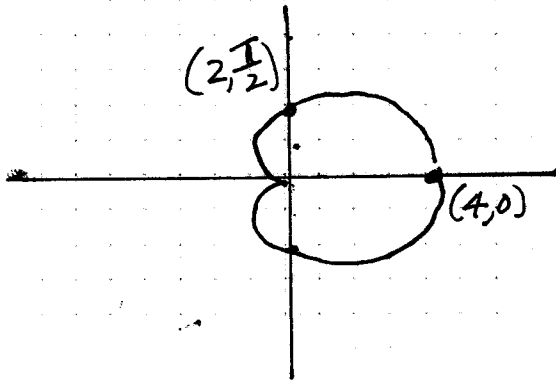
$$r = 4 \cos \theta$$

$$r^2 = 4r \cos \theta$$

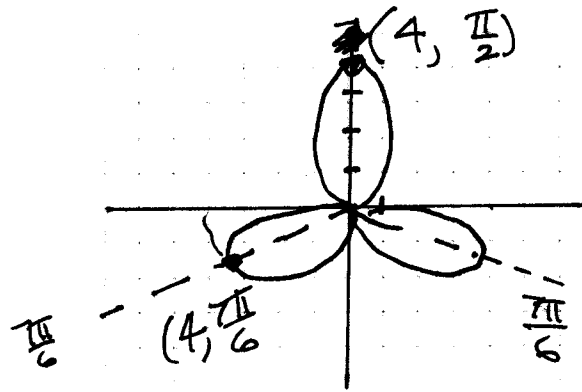
$$x^2 + y^2 = 4x$$

Cardioid

(9) Graph the polar curve: $r = 2 + 2\cos\theta$.



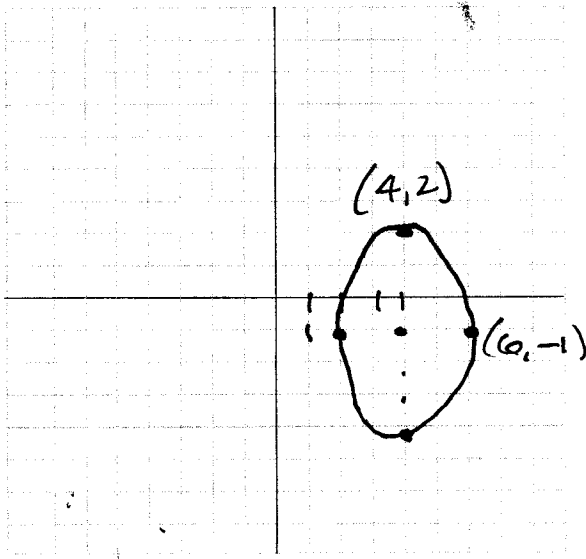
(10) Graph the polar curve: $r = -4\sin 3\theta$.



rose
3-petal
 $\frac{2\pi}{3}$ spacing
length 4
location of one petal
 $4 = -4\sin 3\theta$
 $\sin 3\theta = -1$
 $3\theta = \frac{3\pi}{2}$
 $\theta = \frac{\pi}{2}$

(11) Carefully sketch the graph of $9x^2 + 4y^2 - 72x + 8y + 112 = 0$, and find the following desired information. Label at least 2 points on your graph and show scale. (11 points)

VERTICES: $(4, 2)(4, -4)$ FOCI: $(4, -1 \pm \sqrt{5})$ COVERTICES: $(6, -1)(2, -1)$



Ellipse

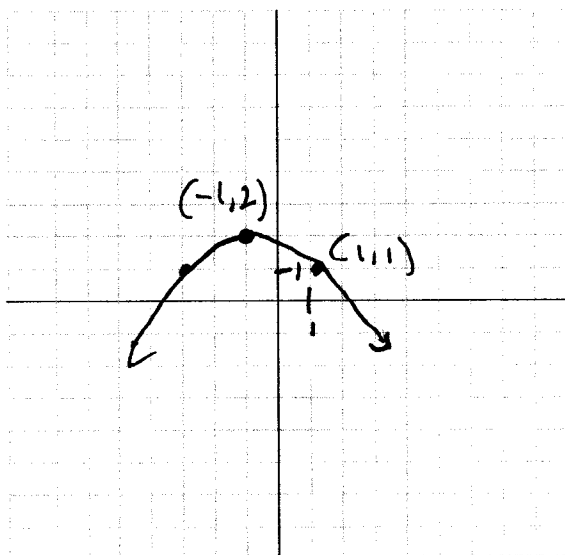
$$\frac{(x-4)^2}{4} + \frac{(y+1)^2}{9} = 1$$

Center $(4, -1)$

Foci: $c^2 = a^2 - b^2 = 9 - 4 = 5$
 $c = \sqrt{5}$

(12) Carefully sketch the graph of $2x^2 + 8y + 4x - 14 = 0$, and find the following desired information. Label at least 2 points on your graph and show scale. (11 points)

VERTEX: $(-1, 2)$ FOCUS: $(-1, 1)$



$$(x+1)^2 = -4(y-2)$$

$$4p = -4$$

$$p = -1$$