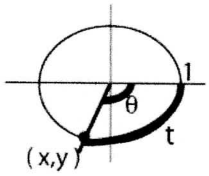
REVIEW: Trigonometry you should know from Math 7A (6.1-6.3, 7.1-7.3)

THE DEFINITIONS OF THE TRIGONOMETRIC FUNCTIONS:

The definitions of the trigonometric functions are given in three different ways, depending on the situation.

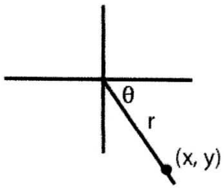
(1) The most general definition allows us to discuss the trig functions as being functions of a ***real number***, not just an angle. Given any real number t, let the point P(x,y) be a corresponding point on the **unit** circle determined by moving a distance of $|t|$ units around the circle starting at the point (1,0) and moving in the counter clockwise direction if t > 0, clockwise if t < 0. The central angle θ corresponding to the real number input t would be an angle of t radians. In this case,

$$\sin t = \sin\theta = y$$
$$\cos t = \cos\theta = x$$
$$\tan t = \tan\theta = \frac{y}{x}$$



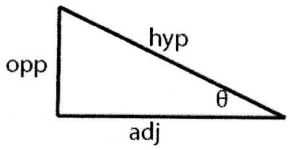
(2) In the case where the point P(x,y) is any point the terminal side of an angle θ in standard position, not necessarily on the unit circle, and **r** is the distance from P to the origin, then

$$\sin\theta = \frac{y}{r}$$
$$\cos\theta = \frac{x}{r}$$
$$\tan\theta = \frac{y}{x}$$



(3) In the special case where θ is an acute angle in a right triangle, the following definitions may be used.

$$\sin\theta = \frac{opp}{hyp}$$
$$\cos\theta = \frac{adj}{hyp}$$
$$\tan\theta = \frac{opp}{adj}$$

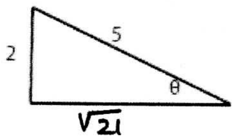


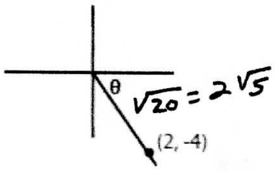
FINDING THE VALUES OF THE TRIG FUNCTIONS

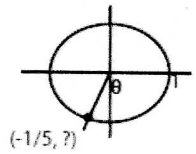
Finding the values of trig functions depends on what information is given.

(1) Given lengths of sides of a right triangle or a point on the terminal side of an angle or a point on the unit circle, we use the appropriate definition.

<u>example</u>: Given the following figures, find:







$\left(-\frac{1}{5}\right)^2 + y^2 = 1$

$y^2 = \frac{24}{25}$

$y = -\frac{2\sqrt{6}}{5}$

(a) cos θ = $\sqrt{21}/5$

(b) csc θ = $5/2$

(c) tan θ = -2

(d) sin θ = $-2/\sqrt{5}$

(e) sin θ = $-2\sqrt{6}/5$

(f) cot θ = $1/2\sqrt{6}$

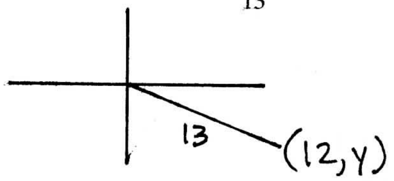
$\frac{-1/5}{-2\sqrt{6}/5}$

FINDING THE VALUES OF THE TRIG FUNCTIONS (cont'd)

(2) Find a desired value of a trig function when given the value of a different trig function at the same input. This can be done in many ways. (See the handout on the "three way problem".)
examples:

(a) Given $\cos\theta = \frac{12}{13}$, θ in Quadrant IV, find $\sin\theta$ $\sec\theta$, $\tan\theta$

$12^2 + y^2 = 13^2$
$y^2 = 25$
$y = -5$

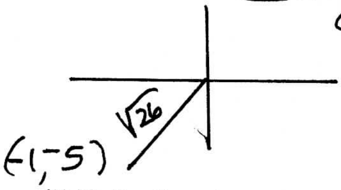


$\sin\theta = \frac{-5}{13}$

$\sec\theta = \frac{13}{12}$

$\tan\theta = \frac{-5}{12}$

(b) Given $\tan\theta = 5$, where $\underbrace{\cos\theta < 0}_{\text{Q III}}$, find $\sin\theta$ $\cos\theta$



$\sin\theta = -5/\sqrt{26}$

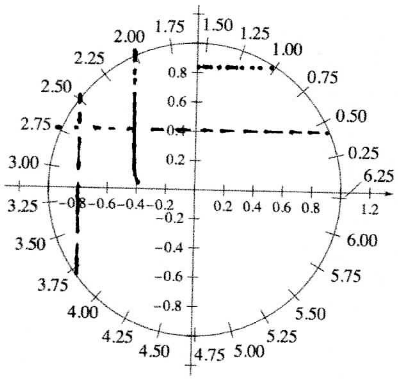
$\cos\theta = -1/\sqrt{26}$

(3) Finding the value of the trig. functions when the angle or real number argument is given. This is the way we are used to finding functional values in algebra. For example, if $f(x) = \sqrt{x}$, we might be asked to find $f(4)$. So if $g(x) = \sin(x)$ for example, how would we find $g(4)$? Unless the argument is one of a handful of special arguments, we will have to approximate it, either using our calculator or by using the "unit circle wrap" idea.

Example: Use the figure to
(a) approximate the value of cos 2 <u>-.4</u> and sin 1 <u>.82</u>

(b) find a value of t such that sint ≈ 0.4 <u>0.4 or 2.75</u>

(c) find a value of t such that cost ≈ -0.8 <u>2.5 or 3.75</u>



FINDING THE VALUES OF THE TRIG FUNCTIONS (cont'd)

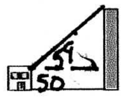
(4) In the special case that the given input is one that we can determine exactly, you should be able to use your knowledge of reference angles and the signs of the trig functions in each quadrant to do these well.

Example: Find each of the following exactly: (no calculator)

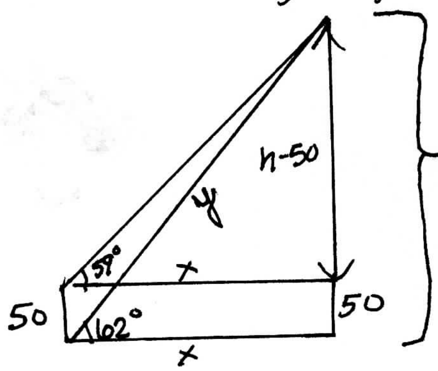
(a) cos (120°) = $-1/2$	(b) sin (3π/2) = -1
(c) sin (-150°) = $-1/2$	(d) cot (- π/4)= -1
(e) tan (225°) = 1	(f) sin (13π/6) = $1/2$
(g) cos 60° = $1/2$	(h) cot 0 = undefined
(i) tan (-30°) = $-1/\sqrt{3}$	(j) cot (2π/3) = $-1/\sqrt{3}$
(k) tan 390° = $1/\sqrt{3}$	(l) sec 7π/6 = $-2/\sqrt{3}$
(m) csc 270° = -1	(n) sin π/4 = $\sqrt{2}/2$
(o) tan (-315°) = 1	(p) cos (-5π/3) = $1/2$
(q) cos 150° = $-\sqrt{3}/2$	(r) sin 7π/3 = $\sqrt{3}/2$
(s) sin 330° = $-1/2$	(t) cot (-3π/4)= 1

Right Triangle Application Problems

Example: If a tower is viewed from the top of a 50 foot building, the angle of elevation to the top of the tower is 59°. If viewed from the ground floor of the same building, the angle of elevation to the top of the tower is 62°. Find the height of the tower. (Show an exact answer and an approximate.)



Using right triangles



Small right triangle

$\frac{h-50}{x} = \tan 59°$

$h-50 = x\tan 59°$

Large right triangle

$\frac{h}{x} = \tan 62°$

$h = x\tan 62°$

Solve system.
Substitute 2nd into 1st

$x\tan 62° - 50 = x\tan 59°$

$-50 = x\tan 59° - \cancel{x}\tan 62°$

$-50 = x(\tan 59° - \tan 62°)$

$x = \frac{-50}{\tan 59° - \tan 62°}$ so $h = \frac{-50}{\tan 59° - \tan 62°}\tan 62°$

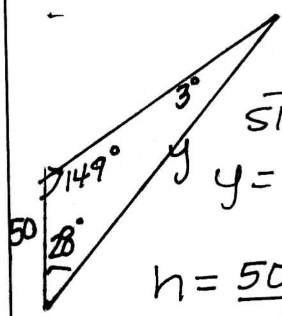
Using Law of Sines...

Using large righ triar

$\sin 62° = \frac{h}{y}$

$h = y\sin 62°$

Find y using oblique t
on left with law of S



$\frac{y}{\sin 149°} = \frac{50}{\sin 3°}$

$y = \frac{50\sin 149°}{\sin 3°}$

$h = \frac{50\sin 149°}{\sin 3°}\sin 6$

≈ 434.5 f