Transformations on the Basic Sine and Cosine Graphs

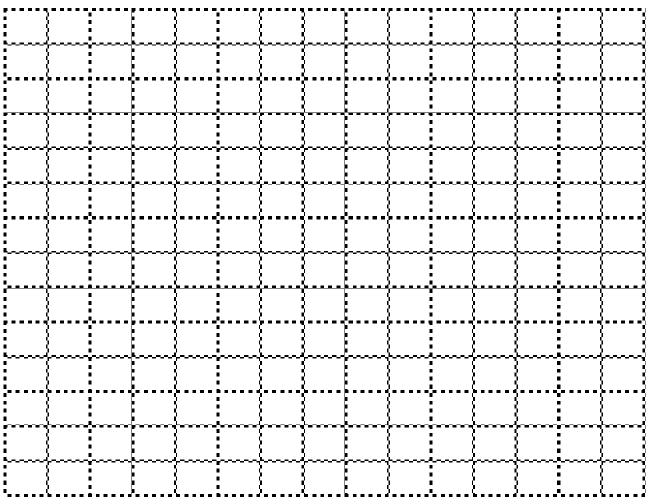
Shift and Reflection

Recall:

VERTICAL SHIFT - For c>0, to graph
f(x) + c, shift the graph of f(x) UP c units,
f(x) - c, shift the graph of f(x) DOWN c units.
HORIZONTAL SHIFT - For c>0, to graph
f(x+c), shift the graph of f(x) LEFT c units,
f(x-c), shift the graph of f(x) RIGHT c units.
REFLECTION -To graph
- f(x), reflect the graph of f(x) in the x axis,
f(-x), reflect the graph of f(x) in the y axis.

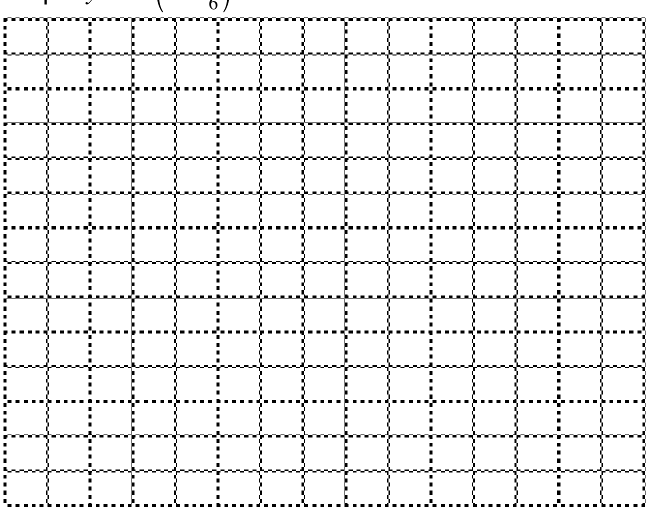
Use the graph of y = sinx below and the principles of transformations to graph

(a) $y = \sin\left(x + \frac{\pi}{4}\right)$ (b) $y = \sin\left(x + \frac{\pi}{4}\right) + 2$ (c) $y = -\sin\left(x + \frac{\pi}{4}\right)$



Sometimes to accommodate the shift, a better choice of <u>scale</u> should be made.

Graph $y = \sin\left(x + \frac{\pi}{6}\right)$



Vertical Stretch/Shrink

Recall:

VERTICAL STRETCH/SHRINK -
cf(x) where c>1, vertically stretch the graph of f(x),
cf(x) where 0< c<1, vertically shrink or compress the graph of f(x).

Use the graph of y = cosx below and the principles of transformations to graph
(a) y = 3cos(x) (b) y= $y = -2\cos\left(x - \frac{3\pi}{4}\right)$

Horizontal Stretch/Shrink

HORIZONTAL STRETCH/SHRINK -
f(cx) where c>1, vertically shrink the graph of f(x),
f(cx) where 0< c<1, vertically stretch the graph of f(x).

Use the graph of y = cosx below and the principles of transformations to graph y= $\cos\left(\frac{1}{2}x\right)$.

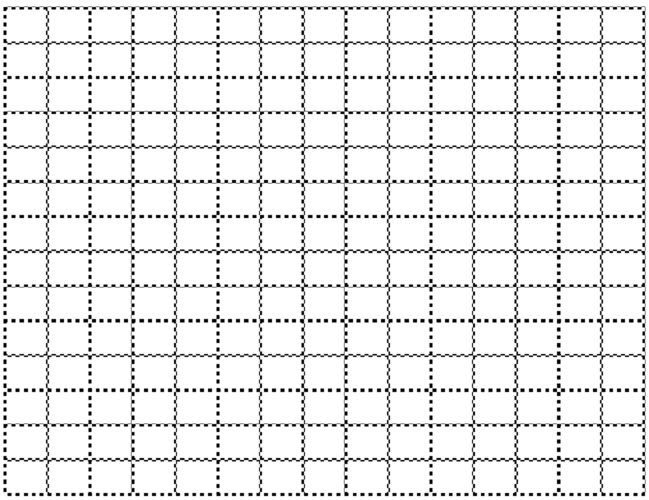
Notice that the period is changed to _____________.

In general, the graph of y = sin (bx) or y = cos(bx) has period = $\frac{2\pi}{|b|}$.

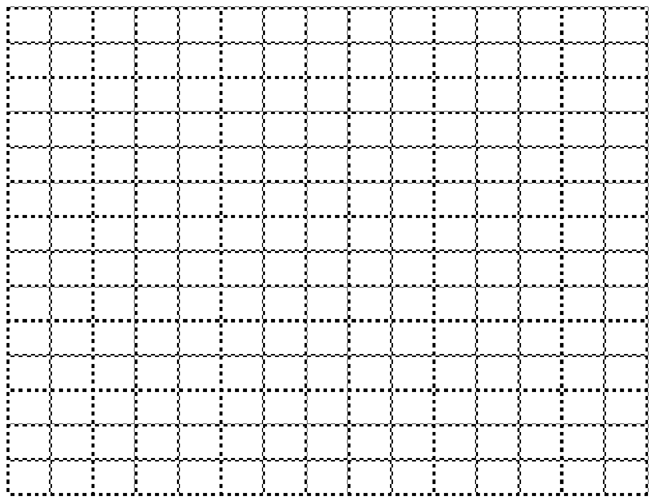
Rather than graph the original sin or cos graph and stretch/compress it, we use the new period to adjust the scale on the x axis (one quarter period is a good first choice of scale) and follow the high-zero-low-zero-high-zero pattern every quarter period.

Try graphing…

y = cos (3x)

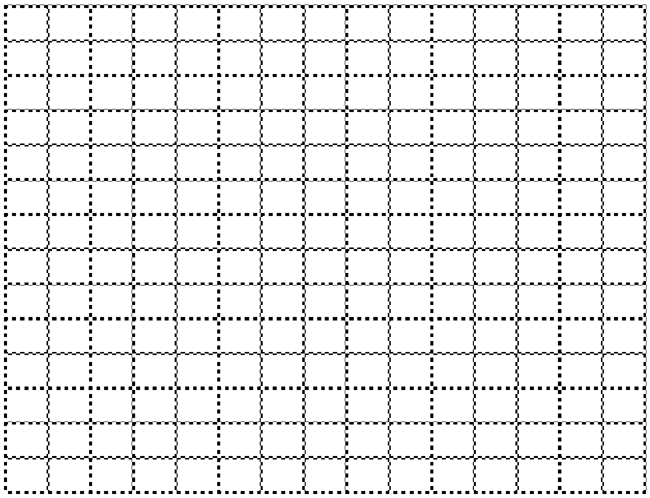


y = 2 sin(2πx)



<u>Combining Horizontal Stretch/Shrink (change of period) with horizontal shift.</u>

Graph f(x) = 3 sin(2x)



Use the graph above to graph $f\left(x+\frac{\pi}{8}\right)$. What transformation is involved?

Find and simplify $f\left(x+\frac{\pi}{8}\right)$

Putting it together: Graphing y= a sin(bx+c) and y= a cos(bx+c)

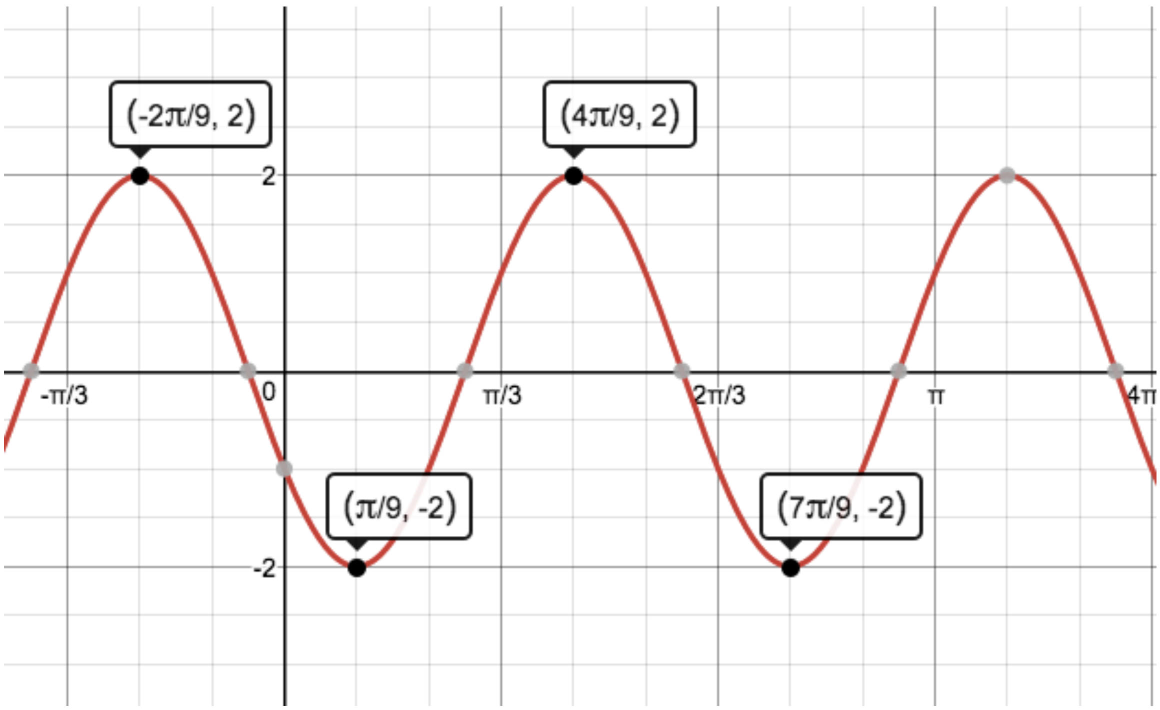
Given an equation of the form y= a sin(bx+c), factor our the b to get $y = a\sin\left(b\left(x + \frac{c}{b}\right)\right)$. In this form we can see that our graph is just the graph of y=a sin(bx) shifted horizontally). The amplitude is $|a|$ and the period is $2\pi/|b|$. The shift is c/b.

Graph: $y = 5\cos\left(3x - \frac{\pi}{4}\right)$ over one period. Show your scale. Label high and low points

Graph: $y = 4\sin\left(\pi x + \frac{\pi}{6}\right)$ over one period. Show your scale. Label high and low points

Using a graph to find the equation.

Find an equation corresponding the graph below. For one of the labeled points, check that it satisfies your equation.



Measure the amplitude, half the distance from the lowest point to the highest. This is a.

Measure the period. Use this to get b since $2\pi/b$ is the period.

Now to finish, we need to find the c of y=a sin(bx+c) or y=a cos(bx+c). To do this, think of the factored form $y = a\sin\left(b\left(x+\frac{c}{b}\right)\right)$ or $y = a\cos\left(b\left(x+\frac{c}{b}\right)\right)$. Read the shift from the graph. There are many possible answers depending on whether you are picturing it as a shift of the cosine graph or the sine graph. Put the shift into the factored form of the equation.